Methods for Modeling Regret of Parameterization Choices

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Introduction

- Traditionally a lot of focus has been placed on **regret of parameter value uncertainty** [1]
- Recently, work has been conducted into understanding of **importance of parameter choice difference on decisions** [2]
- **Stakeholder interests** can vastly change modeling priorities [3]
- **Conflicts of interest** provide interesting fronts on which to **consider decision sets** [4]
- Regret allows us to understand how decisions can be evaluated under different scenarios [5]
- Through a model applied to park location decisions, findings can become more actionable for decision makers

Methods

Sets

L – set of Resident Locations
K – set of Park Locations
I - set of Indices

Decision Variables

y_{ki} - 1 if park $k \in K$ is chosen by Index $i \in I$; 0 otherwise
x_k - 1 if park $k \in K$ is chosen by follower; 0 otherwise
R - max regret of follower choices across all indices $i \in I$

Parameters

q_l^i - aggregate demographic emphasis of resident location $l \in L$ in index $i \in I$
n_l - number of residents in resident location $l \in L$
v_{kl} - value of park location $k \in K$ for resident locations $l \in L$
s_{kl} - 1 if park location $k \in K$ is the primary park of resident location $l \in L$
o_i^* - the perfect foresight choice of the follower within index $i \in I$

- Park Priority described as function of aggregate parameter and primary park zone [6] [7]

$$p_k^i = \sum_{l \in L} [(q_l^i n_l (1 - v_{kl})) s_{kl}]$$

Index Payoff

$p_{i=EJI}^{i'=TPL}$	$p_{i=TPL}^{i'=SVI}$	$p_{i=EJI}^{i'=EJI}$
$p_{i=SVI}^{i'=TPL}$	$p_{i=SVI}^{i'=SVI}$	$p_{i=SVI}^{i'=EJI}$
$p_{i=TPL}^{i'=TPL}$	$p_{i=TPL}^{i'=SVI}$	$p_{i=TPL}^{i'=EJI}$

Decision vectors y_{ki} based on priorities in index $i \in I$ are evaluated in index $i' \in I$

Index Regret

$p_{i=TPL}^{i'=TPL}$-$p_{i=EJI}^{i'=TPL}$	$p_{i=SVI}^{i'=SVI}$-$p_{i=EJI}^{i'=SVI}$	0
$p_{i=TPL}^{i'=TPL}$-$p_{i=SVI}^{i'=TPL}$	0	$p_{i=EJI}^{i'=EJI}$-$p_{i=SVI}^{i'=EJI}$
0	$p_{i'=SVI}^{i'=SVI} - p_{i=TPL}^{i'=SVI}$	$p_{i=EJI}^{i'=EJI}$-$p_{i=TPL}^{i'=EJI}$

Index Regret is defined as the difference between the priority of the best decision vector for a given index $i' \in I$ and a given decision vector

Two Stage Knapsack/Index Regret

- **In the first stage, one decision vector is solved per index to maximize value in that index subject to budget**
- **In the second stage, Index Regret is calculated, and a decision vector is selected from first stage which minimizes regret in worst case among indices assumed correct**

$$\max_{y_{ki} \in \{0,1\}} \sum_{k \in K} p_k^{i'} y_{ki}$$

Subject to:

$$\sum_{k \in K} c_k y_{ki} \le b$$

$$\min \quad \max_i R_i$$

Subject to:

$$R_i \ge \sum_{k \in K} p_k^{i'} y_{ki'} - \sum_{k \in K} p_k^{i'} y_{ki}$$

Bilevel Optimization for Regret of Discrete Decisions

- **Leaders defined by indexes making choices on which items not chosen by follower are most important to them**
- **Follower makes item choices based on minimization of maximum regret faced by any leader**

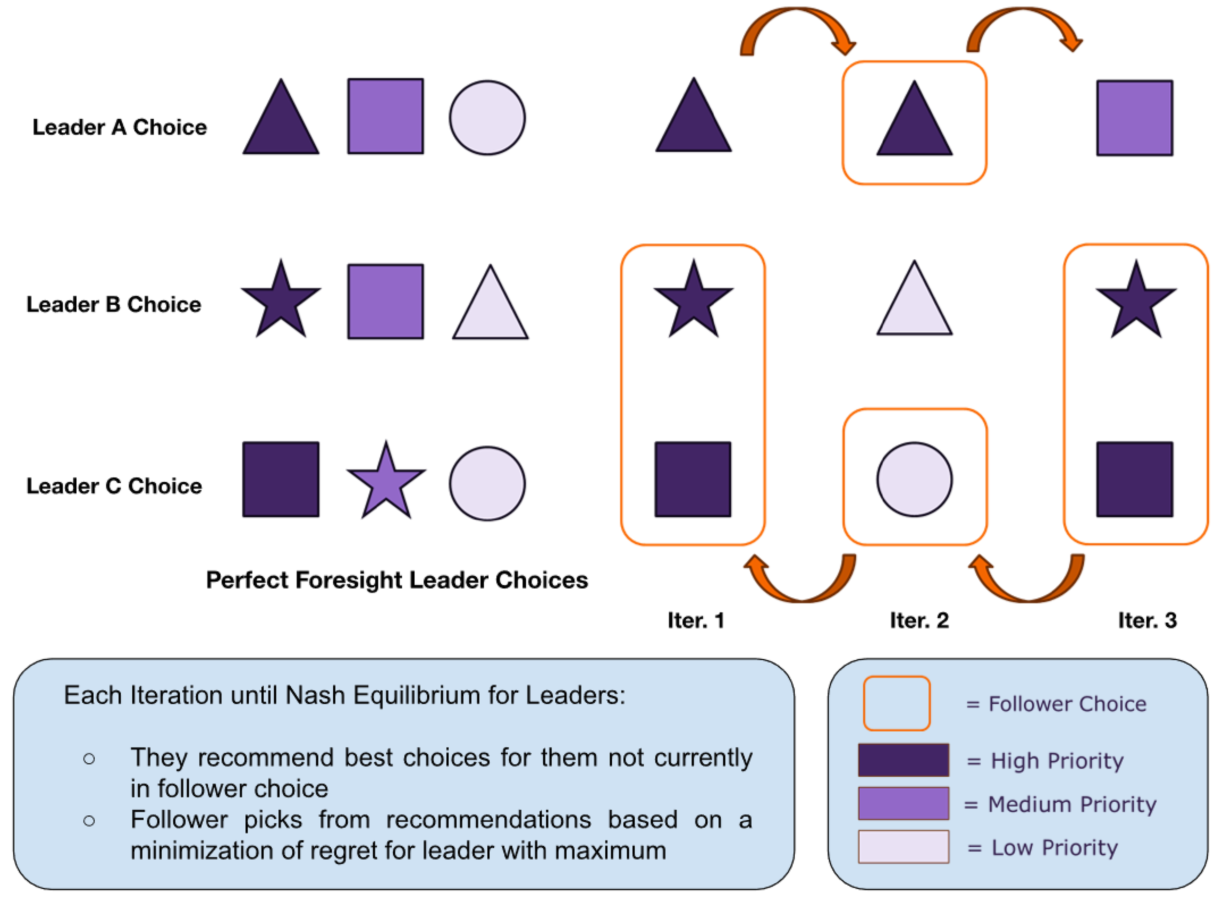
$$\max \quad \sum_{k \in K} p_k^i [y_{ki}(1 - x_k)]$$

$$o_i^* = max \sum_{k \in K} p_k^i x_k$$

$$x_k \in \arg\min \quad R$$

Subject to:

$$R \ge o_i^* - \sum_{k \in K} p_k^i x_k$$



Representation of Bi-Level Stackelberg Game for Mini-Max Regret

Bilevel model is solved using a Big-M and KKT transformation with optimality cuts to ensure that all linear solves are feasible within the true bilevel problem [8] [9] [10]

Results

Two Stage Results

In the graph of normalized index regret vs budget (fig. 1), we see lots of large spikes at cases where decisions are made that differentiate the decision sets. However, most of those peaks are shallow as they quickly normalize again.

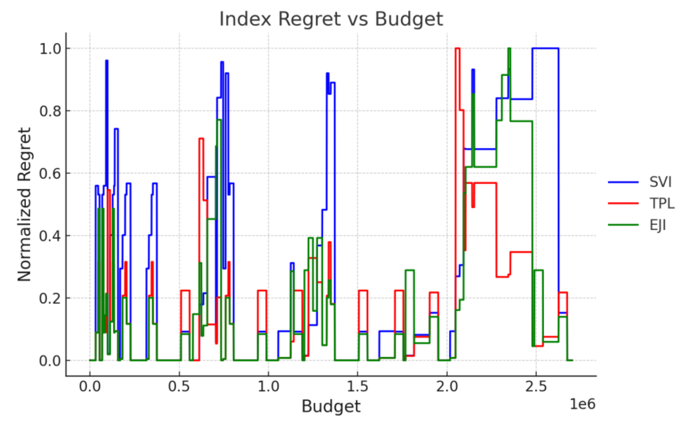


Fig. 1

Bilevel Results

- We see in figure 2 that as more parks are chosen, regret peaks at about half of regret additive parks.
- Figure 3 shows that same peak is at nearly a sixth of those same parks' cost.

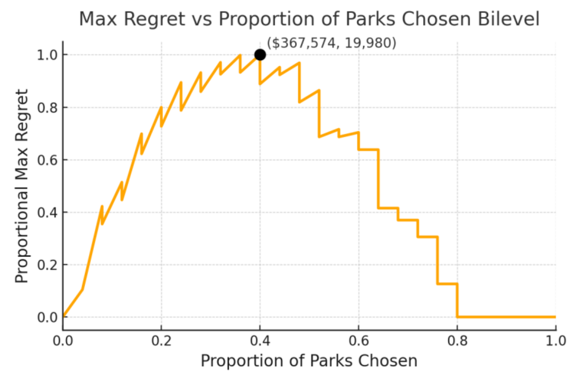


Fig. 2

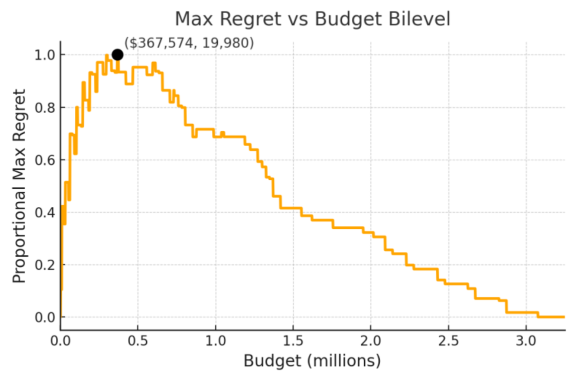


Fig. 3

Analysis & Discussion

- **Index Regret peaks as decisions differ among indices**
- **Furthermore, we cannot recommend a single indices choices without knowledge of the budget**
- **When decisions can be made with knowledge of index recommendations, regret can be expected to generally decrease once decision sets can only become more similar, even if those parks are the least expensive**

Conclusion

- Through **understanding regret** of how parameter choices best describe a system, decision makers can better understand the **risks of misidentifying stakeholder needs**
- **Relationships between recommendations and decisions** can be **modeled** to make more **actionable decisions** in the context of regret

References

1. Xu, D. and S. L. Albin (2003). "Robust Optimization of Experimentally Derived Objective Functions." IIE Transactions **35**(9): 793-802.
2. Kujala, H., et al. (2018). "Not all data are equal: Influence of data type and amount in spatial conservation prioritization." Methods in Ecology and Evolution **9**(11): 2249-2261.
3. O'Brien, T. C., et al. (2025). "Developing optimization models with cognitive systems engineering." Journal of the Operational Research Society.
4. Caparros-Midwood, D., et al. (2015). "Optimized spatial planning to meet long term urban sustainability objectives." Computers, Environment and Urban Systems **54**: 154-164.
5. Audibert, J.-Y., et al. (2014). "Regret in Online Combinatorial Optimization." Mathematics of Operations Research **39(1): 31-45.**
6. Molano Losada, A. (2008). Modelo de localización de áreas urbanas para construir nuevos parques vecinales en Bogotá. [online] Uniandes. [Acceso : 24 de julio 2025]
7. Young, A., et al. (2022/12/08). "An Optimization Approach to Improve Equitable Access to Local Parks."
8. Hansen, P., et al. (1992). "New Branch-and-Bound Rules for Linear Bilevel Programming." Siam Journal on Scientific and Statistical Computing **13**(5): 1194-1217.
9. Fischetti, M. and M. Monaci (2020). "A branch-and-cut algorithm for Mixed-Integer Bilinear Programming." European Journal of Operational Research **282(2): 506-514.**
10. Codato, G. and M. Fischetti (2006). "Combinatorial benders' cuts for mixed-integer linear programming." Operations Research **54(4): 756-766.**
11. Gabriel, S. A. and F. U. Leuthold (2010). "Solving discretely-constrained MPEC problems with applications in electric power markets." Energy Economics **32**(1): 3-14.
12. Luna, J. P., et al. (2023). "Regularized Equilibrium Problems with Equilibrium Constraints with Application to Energy Markets." Siam Journal on Optimization **33**(3): 1767-1796.

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